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HYDROMAGNETIC WAVES IN A PLASMA WITH FINITE LARMOR RADIUS

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ABSTRACT

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The effect of finite Larmor radius and Larmor frequency on hydromagnetic waves in a plasma is investigated. It is concluded that the finite Larmor radius has considerable influence. A uniform rotation is also included in view of its astrophysical importance.

Author

1. INTRODUCTION

The purpose of the present paper is to investigate the effect of finite ion Larmor radius on the hydromagnetic wave propagation in a plasma. The well-known results regarding Alfven waves were obtained by using idealized hydromagnetic equations which are, strictly speaking, valid only in the limit that theLarmor radii of the charged particles (electrons and protons) are effectively zero and the corresponding Larmor frequencies regarded as infinitely large. In many astrophysical situations like the Solar Corona, interplanetary and interstellar plasmas, it is known that the approximation (zero Larmor radius and infinite Larmor frequency) is not valid. It is, therefore, interesting to study the modifications in the hydromagnetic wave propagation if one relaxes the above mentioned approximation. It may be mentioned that Rosenbluth et.al.[1] have found, in connection with the gravitational instability of a magnetized plasma, that the 'flute' instability is reasonably suppressed by finite Larmor radius effects, particularly for small wave-length perturbations propagating normal to the ambient magnetic field. Their approach is based on the collisionless Boltzmann equation. Roberts and Taylor[2] and more recently Rosenbluth and Simon[3] have shown how equivalent results can be obtained using hydromagnetic equations modified to take account of the finite ion Larmor radius. These investigations are, however, restricted to low β plasmas so that the prevailing magnetic field does not change during the course of perturbations. The velocity vector was also confined to a plane normal to the ambient magnetic field. Hydromagnetic wave propagation is, therefore, not included in their investigations in view of the above mentioned restrictions.

2. BASIC EQUATIONS AND DISPERSION RELATION

Consider a homogenous, unbounded, collisional plasma having N_0 electrons per c.c. and an equal number of protons. The electron and ion temperatures are assumed equal, and the plasma, in the absence of a prevailing magnetic field, is characterized by an isotropic pressure $p_0 \; (= 2N_0 kT)$, K being the Boltzmann constant and T, the temperature of the medium. The plasma pressure is rendered anisotropic owing to the ambient magnetic field, and the anisotropicity is determined by the Larmor frequency, the Larmor radius, and the macroscopic velocity gradients. As the initial state under consideration is static (or partaking in a uniform rotation (§3)), the plasma pressure is anisotropic only in the perturbed state of plasma. Again we may regard the electron pressure to retain isotropic character even during perturbation, on account of the negligible electron Larmor radius compared to the ion Larmor radius. We will regard the plasma to be non-heat conducting and having an isotropic electric conductivity σ. It may be remarked that the heat conduction would arise only in the perturbed state as the initial configuration is devoid of temperature gradients and electric field. Even during perturbation heat flow can be reasonably neglected as one can show easily that the various coefficients of the heat flow vector (Bernstein and Trehan[4] equation II - (71)) are very much smaller compared to the electrical conductivity of the medium.

The basic equations are written as,

$$\rho \frac{d\underline{v}}{dt} = -\nabla p - \nabla\cdot \overleftrightarrow{\Pi} + \frac{\underline{j}\times\underline{B}}{c} \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) = 0 \tag{2}$$

$$\nabla \times \underline{E} = -\frac{1}{c}\frac{\partial \underline{B}}{\partial t} \tag{3}$$

$$\nabla \cdot \underline{B} = 0 \tag{4}$$

$$\nabla \cdot \underline{E} = 0 \tag{5}$$

$$\nabla \times \underline{B} = \frac{4\pi}{c}\underline{j} \tag{6}$$

$$\frac{dp}{dt} = -\gamma p \nabla \cdot \underline{v} - \frac{2}{3}\Pi : \nabla \underline{v} + \frac{2}{3}\underline{j} \cdot \left(\underline{E} + \frac{\underline{v} \times \underline{B}}{c}\right) \tag{7}$$

and

$$\underline{E} + \frac{\underline{v} \times \underline{B}}{c} - \frac{\underline{j} \times \underline{B}}{Nec} - \frac{\underline{j}}{\sigma} + \frac{1}{Ne}\nabla p_e = 0 \tag{8}$$

Here $\rho(=MN)$, $p(=p_i+p_e)$ and $\overleftrightarrow{\Pi}$ denote the material density, and the scalar and tensorial part of the plasma pressure respectively. Equation (7) is the usual adiabatic equation of state with $\gamma = 5/3$, as modified to take account of a net electric field and the tensorial pressure in the plasma. The final equation (8) is the 'generalized Ohm's law' without the electron inertia term $-\frac{m_e}{Ne^2}\frac{\partial}{\partial t}\dot{\underline{J}}$, which is left out as being negligible because we are dealing with frequencies well below the plasma frequency. It may be noted that in view of the unperturbed state being devoid of currents and velocity, the last two terms on the right side of equation (7) can be omitted in a linearized analysis under investigation, as being of second order of smallness.

Taking the ambient uniform field B_0 along the x-axis and assuming the perturbations to vary as exp. $[nt + ik_x x + ik_y y]$, we may write the perturbation equations as,

$$\rho_0 n u_x = -ik_x \delta p - (ik_x \Pi_{xx} + ik_y \Pi_{yy}) \tag{9}$$

$$\rho_0 n u_y = -ik_y \delta p - (ik_x \Pi_{xy} + ik_y \Pi_{yy}) + \frac{B_0}{4\pi}(ik_x b_y - ik_y b_x) \tag{10}$$

$$\rho_0 n u_z = -(ik_x \Pi_{xz} + ik_y \Pi_{yz}) + \frac{B_0}{4\pi} ik_x b_z \tag{11}$$

$$n\delta\rho = -\rho_0 \nabla\cdot\underline{u} \tag{12}$$

$$n\delta p = -\gamma p_0 \nabla.\underline{u} \tag{13}$$

$$(n+\eta k^2) b_x - \frac{cB_0}{4\pi N_0 e} k_x k_y b_z = -B_0 \nabla.\underline{u} + B_0 i k_x u_x \tag{14}$$

$$(n+\eta k^2) b_y + \frac{cB_0}{4\pi N_0 e} k_x^2 b_z = B_0 i k_x u_y \tag{15}$$

$$(n+\eta k^2) b_z + \frac{cB_0}{4\pi N_0 e} i k_x (i k_x b_y - i k_y b_x) = B_0 i k_x u_z \tag{16}$$

and

$$i k_x b_x + i k_y b_y = 0 \tag{17}$$

Here (u_x, u_y, u_z) and (b_x, b_y, b_z) denote the compoenents of the perturbations $\underline{u}$ and $\underline{b}$ in velocity and magnetic field vectors respectively. $\delta\rho$, δp denote perturbations in density and pressur e and $\eta \left(= 1/4\pi\sigma\right)$ stands for the'magnetic viscosity' of the medium. The equilibrium quantities are suffixed '0'. In writing equations (14)--(16) use has been made of equations (3), (6) and (8). The ion pressure tensor components Π_{xx} etc. ion the equations (9)--(11) are written as, (Thompson[5])

$$\Pi_{xx} = -\frac{p_0\tau}{9}\left(2ik_x u_x - ik_y u_y\right) \tag{18}$$

$$\Pi_{xy} = \Pi_{yx} = -\frac{p_0 ik}{4\omega} u_z \tag{19}$$

$$\Pi_{yy} = -\frac{p_0\tau}{18}\left(ik_y u_y - 2ik_x u_x\right) - \frac{p_0}{8} ik_y u_z \tag{20}$$

$$\Pi_{yz} = \Pi_{zy} = \frac{p_0 ik_y}{8\omega} u_y \tag{21}$$

$$\Pi_{xz} = \Pi_{zx} = \frac{p_0}{4\omega}\left(ik_x u_y + ik_y u_x\right) \tag{22}$$

Here τ denotes the ion - ion collision time and $\omega\left(=\frac{eB_0}{Mc}\right)$ the ion Larmor frequency.

Using equations (12), and (18)--(22) we may rewrite the equations (9)--(11) as,

$$u_x\left[1 + \frac{k_x^2 S^2}{n^2} + \frac{2}{9}\frac{k_x^2 p_0}{\rho_0 n}\tau\right] = -k_x k_y u_y\left[\frac{S^2}{n^2} - \frac{p_0\tau}{9n\rho_0}\right] - k_x k_y \frac{p_0}{4\omega\rho_0 n} u_z \tag{23}$$

$$u_y\left[1 + \frac{k_y^2 S^2}{n^2} + \frac{k_y^2 p_0\tau}{18 n\rho_0}\right] = -k_x k_y u_y\left[\frac{S^2}{n^2} - \frac{p_0\tau}{9n\rho_0}\right] - p_0\frac{u_z}{8n\rho_0\omega}\left(k_y^2 + 2k_x^2\right)$$
$$+ \frac{B_0}{4\pi\rho n}\left(ik_x b_y - ik_y b_x\right) \tag{24}$$

and

$$u_z = \frac{k_x k_y p_0}{4n\rho_0\omega} u_x + \frac{p_0}{8n\rho_0\omega}\left(2k_x^2 + k_y^2\right) u_y + \frac{ik_x B_0}{4\pi\rho_0 n} b_z \qquad (25)$$

Here S^2 is written for $\gamma p_0/\rho_0$, (S being the sound speed for the medium).

We may now derive the dispersion relation by using equations (14)--(17) and (23)--(25). After some simplifications we obtain the following dispersion equation.

$$\left[\left\{X + \left(\frac{p_0}{8n\rho_0\omega}\right)^2\left(k_y^2 + 2k_x^2\right)\right\}Y - k_x^2 k_y^2\left\{Z + 2\left(\frac{p_0}{8n\rho_0\omega}\right)^2\left(k_y^2 + 2k_x^2\right)\right\}^2\right]$$

$$\cdot\left[\left\{1 + \frac{k_x^2 V_0^2}{nn_1} + \left(\frac{cB_0}{4\pi N_0 e}\right)^2 \frac{k_x^2 k^2}{n_1^2}\right\}Y - \frac{B_0^2}{\pi\rho_0 n n_1}\left(\frac{p_0}{8n\rho_0\omega}\right)^2 k_x^4 k_y^2\right]$$

$$= \frac{k_x^2 V_0^2}{nn_1}\left[\left\{\frac{p_0}{8n\rho_0\omega}\left(k_y^2 + 2k_x^2\right) + \frac{cB_0}{4\pi N_0 e}\frac{k^2}{n_1}\right\}Y - 2k_x^2 k_y^2\left(\frac{p_0}{8n\rho_0\omega}\right)\left\{Z + 2\left(\frac{p_0}{8n\rho_0\omega}\right)^2\left(k_y^2 + 2k_x^2\right)\right\}\right]^2 \qquad (26)$$

where

$$X = 1 + \frac{k_y^2 S^2}{n^2} + \frac{k^2 V_0^2}{nn_1} + \frac{k_y^2 p_0 \tau}{18 n\rho_0} \qquad (27)$$

$$Y = 1 + \frac{k_x^2 S^2}{n^2} + \frac{2}{9}\frac{k_x^2 p_0}{n\rho_0}\tau + \left(\frac{k_x k_y p_0}{4\rho_0 n\omega}\right)^2 \qquad (28)$$

$$Z = \frac{S^2}{n^2} - \frac{p_0 \tau}{9 n \rho_0} \tag{29}$$

$$k^2 = k_x^2 + k_y^2 \tag{30}$$

$$V_0^2 = \frac{B_0^2}{4\pi\rho_0} \tag{31}$$

$$n_1 = n + \eta k^2 \tag{32}$$

The dispersion relation (26) is rather complicated for general discussion including finite conductivity, finite Larmor radius and Hall current term. We will, therefore, discuss some special cases for an infinitely conducting plasma.

Perpendicular Propagation $k_x = 0, \ k_y = k$.

For propagation normal to the ambient magnetic field, the dispersion relation gives,

$$U_p^2 = (V_0^2 + S^2) + \left(\frac{p_0 k}{8\rho_0 \omega}\right)^2 - \frac{i k U_p U_0}{18 \rho_0} \tag{33}$$

where Up denotes the phase velocity in/k. The equation (33) represents a damped, dispersive mode. The phase velocity is effectively increased by the ion-Larmor radius term involving $p_0/\rho_0\omega$ $(= r_L^2 \omega$, r_L being the ion Larmor radius). The Hall current term, however, does not affect the perpendicular node.

Parallel Propagation $k_x = k$, $k_y = 0$.

If the propagation is only confined along the direction of the ambient magnetic field, the dispersion relation (26) gives the following modes. We obtain a sound mode defined by,

$$n = \pm ikS\left[1 - \frac{k^2 p_0^2 \tau^2}{81 \rho_0 \gamma}\right]^{1/2} - \frac{k^2 p_0}{9\rho_0}\tau \tag{34}$$

This represents a sound mode damped due to mutual collisions and independent of the following two modes described by,

$$U_{P_{1,2}}^2 = V_0^2\left[1 + \frac{1}{2}\left(\delta^2 + \zeta^2\right) \mp \left(\delta + \zeta\right)\left\{1 + \frac{1}{4}\left(\delta - \zeta\right)^2\right\}^{1/2}\right] \tag{35}$$

where

$$\delta = \frac{kV_0}{\omega} = \frac{kc}{\omega_p} \qquad (\ \omega_p = \text{ ion Plasma frequency}) \tag{36}$$

and

$$\zeta = \frac{kp_0}{4\omega\rho_0 V_0} = \frac{k^2 r_L^2}{4\delta} \tag{37}$$

The phase velocities Up_1, Up_2 for the two modes, as measured in units of Alfven speed V_0, are plotted in figures 1 and 2 against the parameter (equation (36)) for a few values of the parameter ξ (equation (37)). The values chosen for ξ are 0, 0.2, 0.4, 0.6, and 0.8 corresponding to graphs (a)--(e). The parameters δ and ξ are, as is clear from equations (36) and (37), measures respectively of the wave number of perturbation and Larmor radius (or temperature for a given magnetic field) in a plasma of specified number density of particles. We find that the phase velocity of hydromagnetic wave propagation is considerably modified due to finite values of ion Larmor radius and frequency. There are now two modes, instead of one, and they are dispersive in character. The phase velocity of one mode, corresponding to positive sign in equation (35) is always more than the Alfven speed (fig.1), and the increment is more for increase in Larmor radius (or temperature for a fixed magnetic field) for a fixed wave number of perturbation and vice versa. The other mode (fig. 2) is characterized by a phase speed which is less than the Alfven speed for large wavelengths and small Larmor radius (low temperature) but shows a minimum value for a critical δ for a given value of Larmor radius (parameter ξ). The critical wave number for minimum phase speed depends on the characteristic value for the parameter ξ, but thereafter the phase velocity again increases with increase in δ.

3. EFFECT OF UNIFORM ROTATION

We will now consider the effect of a uniform rotation $\underline{\Omega}\,(\Omega_x, \Omega_y, 0)$ on hydromagnetic waves propagating along the ambient magnetic field in a plasma having finite values of ion Larmor radius and Larmor frequency. For simplicity we shall restrict to incompressible perturbations only. In the presence of uniform rotation the equation (1) contains two additional terms on the right hand side, namely, the centripetal force $-\rho\,\underline{\Omega} \times (\underline{\Omega} \times \underline{r})$ and the coriolis force $2\rho\,(\underline{v} \times \underline{\Omega})$. The equations (2)--(6) are unmodified except that $\nabla \cdot \underline{v} = 0$. The 'generalized' Ohm's law should contain a term $\frac{2m_e}{Ne^2}(\underline{j} \times \underline{\Omega})$ which is, however, neglected as being small in comparison to other terms. Strictly speaking, there is an initial centripetal force which is left unbalanced in the unperturbed state. We may, therefore, restrict to small scale lengths in a slowly rotating configuration so that the centripetal force could be neglected. It may be remarked that the centripetal force does not affect the stability analysis as it remains unperturbed. With the above mentioned modifications we can derive the dispersion relation proceeding as in the non-rotating case. For propagation parallel to the prevailing magnetic field in a dissipative plasma, the dispersion relation is written as,

$$\left[n n_1 + k^2 V_0^2 + \frac{n_1}{n}\left(\frac{k^2 p_0}{4\omega\rho_0} - 2\Omega_x\right)^2\right]\left[n n_1 + k^2 V_0^2 + \frac{n}{n_1}\frac{k^4 V_0^4}{\omega^2}\right] = \frac{k^2 V_0^2}{n n_1}\left[\left(\frac{k^2 p_0}{4\omega\rho_0} - 2\Omega_x\right) n_1 + \frac{k^2 V_0^2}{\omega} n\right]^2 \qquad (38)$$

where the symbols have the same meanings as in equation (26). It is worth noting that the component of rotation vector normal to the ambient magnetic field does not contribute, and the parallel component of rotation vector appears along with the finite Larmor radius term. Thus the effect of rotation is to effectively decrease the Larmor radius by an amount depending upon the wave number of perturbation and the Larmor frequency. The dispersion relation (38) reduces to that obtained by Lehnert[6] for a rotating plasma when the Hall current term and the Larmor radius terms are left out.

For infinitely conducting plasma, the phase velocities of the hydromagnetic modes in a uniformly rotating medium are given by an equation exactly similar to equation (35) except that the term $kp_0/4\omega\rho_0 V_0$ is replaced by $\left(\frac{kp_0}{4\omega\rho_0 V_0} - \frac{2\Omega_x}{kV_0}\right)$ everywhere in the equation. The curves (a)--)i) in figures 1 and 2 show also the effect of rotation on hydromagnetic waves in a plasma with finite Larmor radius and the Hall current. The curves for positive and negative values of $\zeta_{eff.}\left[=\left(\frac{kp_0}{4\omega\rho_0 V_0} - \frac{2\Omega_x}{kV_0}\right)\right]$ correspond respectively to the situation where the finite Larmor radius contribution exceeds that due to uniform rotation and vice versa.

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CAPTIONS

Figure 1 The phase velocity U_{p1} (in units of Alfven speed) is plotted against $\delta \left(= \frac{kc}{\omega_p}\right)$ for various values of $\xi_{eff} \left[= \left(\frac{k p_0}{4 \omega \rho_0 V_0} - \frac{2\Omega_x}{k V_0}\right)\right]$.

Figure 2 The phase velocity U_{p2} (in units of Alfven speed) is plotted against $\delta \left(= \frac{kc}{\omega_p}\right)$ for various values of $\xi_{eff} \left[= \left(\frac{k p_0}{4 \omega \rho_0 V_0} - \frac{2\Omega_x}{k V_0}\right)\right]$.

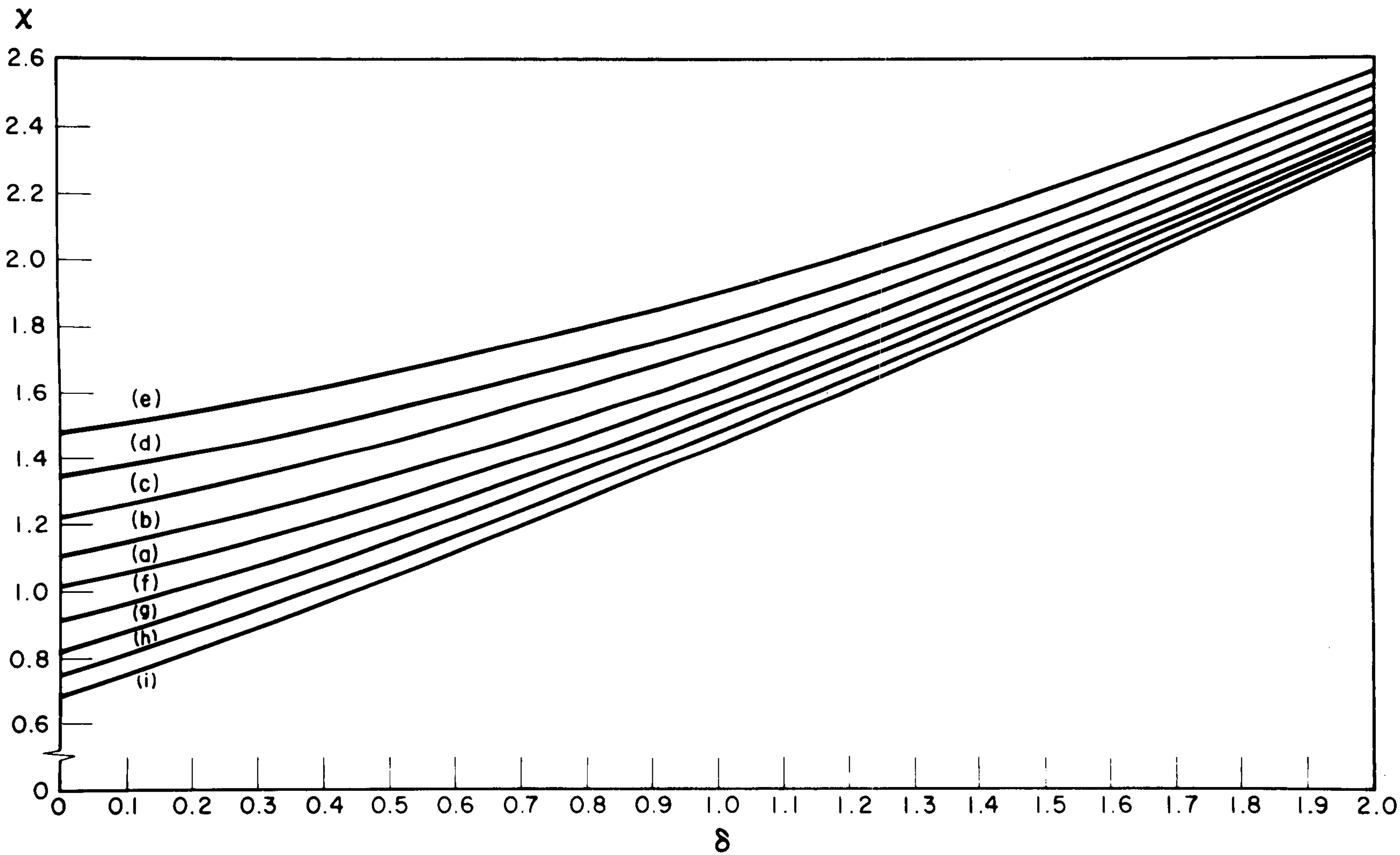
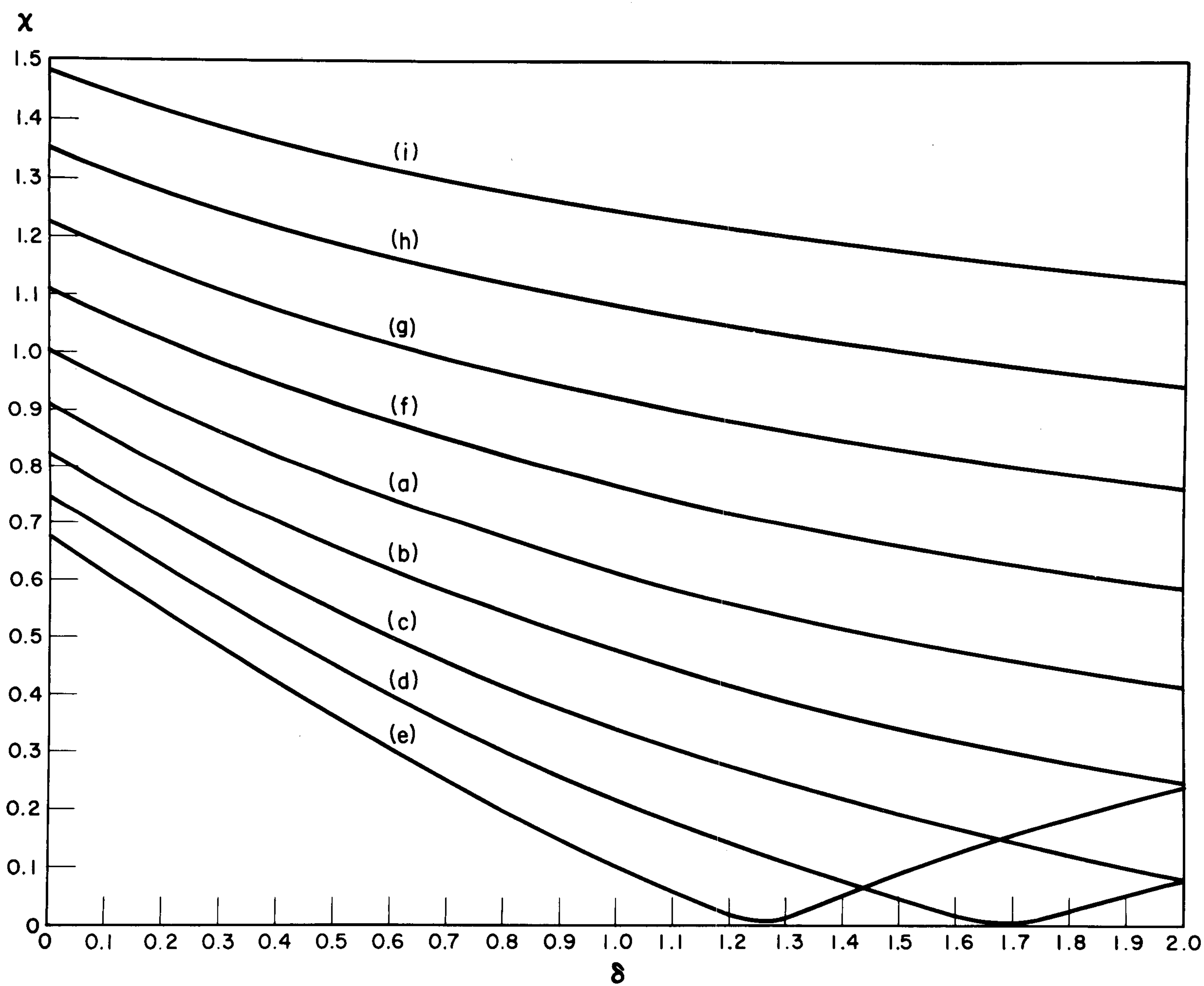

χ
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0
(e)
(d)
(c)
(b)
(a)
(f)
(g)
(h)
(i)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
δ

Figure 1


χ
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1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
(i)
(h)
(g)
(f)
(a)
(b)
(c)
(d)
(e)
0.1
0.2
0.3
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0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
δ

Figure 2